 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – **APRIL 2012**

# MT 2813 - ALGORITHMIC GRAPH THEORY

Date : 24-04-2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

Answer all questions. Each question carries 20 marks.

1. (a) Prove that every nontrivial loopless connected graph has at least two vertices that are

not cut vertices. (5)

(OR)

1. Define walk, trail, path, cycle and tree with examples. (5)
2. (i) Prove that a connected graph is a tree if and only if every edge is a cut edge.

(ii) State Dijkstra’s algorithm and use it to find the shortest distance between the vertices *A* and *H* in the weighted graph given below. (5+10)



(OR)

1. (i) Find a Hamiltonian path or a Hamiltonian cycle if it exists in each of the graphs given below. If it does not exist, explain why?



1. Prove that an edge *e* of *G* is a cut edge of *G* if and only if *e* is contained in no cycle of G. (6+9)
2. (a) Define the connectivity and edge connectivity of a graph *G* and give an example of a graph G in which κ(*G*) < *κ*′ (*G*) < *δ* (G). (5)

(OR)

(b) Prove that K5 is non-planar. (5)

(c) Define Eulerian graph. State and prove a necessary and sufficient condition for a graph to be Eulerian. (15)

(OR)

1. Explain Chinese Postman problem. Use Fleury’s algorithm to find the Euler tour for the following graph.



(15)

3. (a) Define and obtain a minimal vertex separator for *b* and *g*.

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(5)

(OR)

(b) State the Lexicographic breadth first search algorithm. (5)

(c) Let *G* be an undirected graph. Then prove that the following statements are equivalent.

(i) *G* is triangulated.

(ii) *G* is the intersection graph of a family of subtrees of a tree.

(iii) There is a tree *T* whose vertex set is the set of maximal cliques of a graph *G* such that each induced subgraph: *v* ε *V* is connected, where *Kv* consists of those maximal cliques which contain *v*. . (15)

(OR)

1. (i) Prove that a family of subtrees of a tree satisfies the Helly property.

(ii) Let *G* be an undirected graph. Then prove that the following statements are equivalent.

(1) *G* is triangulated.

(2) Every minimal vertex separator induces a complete subgraph of *G*. (6+9)

4. (a) Let *G* be a split graph with the vertex set partitioned into a stable set *S* and a clique *K*. If |*S*| = α(*G*) and |*K*| = ω(*G*) – 1, then prove that there exists an *x* *ε* S such that K +{*x*} is a clique. (5)

(OR)

(b) Define a permutation graph. Draw the permutation graph corresponding to the permutation [9, 7, 1, 5, 2, 6, 3, 4, 8]. (5)

(c) Let *G* be an undirected graph with degree sequence *d*1 ≥ *d*2 ≥ … ≥ *dn* and let *m* = max {*i* : *di* ≥ *i –* 1}. Then prove that *G* is a split graph if and only if . (15)

(OR)

(d) (i) Prove that an undirected graph *G* is a permutation graph if and only if *G* and  are comparability graphs.

(ii) Obtain the permutation from the following transitive orientations of *G* and.

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(8+7)

5. (a) State the *depth-first search algorithm.* (5)

(OR)

(b) Obtain an interval representation for the following interval graph.

5b.emf

(5)

(c) State the *breadth-first search algorithm* and simulate it on the following graph by

selecting the vertex *a.*



(OR)

(d) Let *G* be an undirected graph. Then prove that the following statements are

equivalent.

(1) *G* is an interval graph

(2) *G* contains no chordless 4-cycle and its complement  is comparability

graph.

(3) The maximal cliques can be linearly ordered such that for every vertex *v* of

*G* the maximal cliques containing *v* occurs consecutively. (15)

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